Reconfiguration of Nonlinear Faulty Systems

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***Abstract*— The problem of reconfiguration in faulty systems containing non-smooth nonlinearities is considered. To solve the problem, the control law is constructed providing insensitive to effects of faults. The suggested solution is based on so-called logic-dynamic approach. The main feature of this approach is that it allows to use well known linear methods for systems with non-smooth nonlinearities. Existing conditions are established and the expression for new control is given. The example illustrates the theoretical results.**

***Keywords:* nonlinear systems, faults, reconfiguration, logic-dynamic approach.**

I. INTRODUCTION

In this paper, the problem of fault tolerant control (FTC) in technical systems of critical purposes is investigated. Two main approaches to the FTC are known. The first one is fault accommodation while the second one involves system reconfiguration [1, 8, 10].

The paper concentrates on the problem of reconfiguration in systems described by nonlinear equations. It is assumed that the faults are detected and isolated by known methods in the system. In our approach, we interpret faults as unknown disturbances. Then the disturbance decoupling problem (DDP) solution is used for solving the plant reconfiguration problem (PRP).

Two solutions of PRP have been proposed. The first one uses the methods of adaptive control and assumes that faults are detected and estimated and then the control law accommodation is designed [1, 2, 10]. The second approach determines a control law such that some function of the system output is full decoupled with respect to effects of faults [6]. Unlike the first approach, the fault estimation is not required in the second approach.

The problem of reconfiguration in nonlinear systems containing non-smooth functions was considered in [4] and then in [8, 11, 12]. The peculiarity of [4] is that the known DDP solution is used for solving the PRP. Besides, a solution is based on so-called algebra of functions and demands sophisticated analytical computations. In contrast to [4], the papers [11, 12] use so-called logic-dynamic approach developed in [13]. The aim of our paper is to find a solution of the PRP by the methods of linear algebra.

The rest of the paper is organized as follows. In Section II, the basic models and problem statement are formulated. The disturbance decoupling problem is solved in Section III. Section IV considers solution of the PRP based on the DDP solution. Illustrative example is given in Section V. Section VI concludes the paper.

II. BASIC MODELS AND PROBLEM STATEMENT

Consider dynamic system

$$\begin{aligned} &x(t+1) = Fx(t) + Gu(t) + Dd(t) + \Psi(x(t), u(t)), \\ &y(t) = Hx(t), \end{aligned} \tag{1}$$

where

$$\Psi(x(t), u(t)) = C\begin{pmatrix} \varphi_1(A_1 x(t), u(t)) \\ \cdots \\ \varphi_q(A_q x(t), u(t)) \end{pmatrix},$$

$x \in R^n$ is the state vector, $u \in R^m$ is the control vector, $y \in R^l$ is the vectors of outputs; F and G are matrices describing the linear dynamic part of the system; H, C, and D are matrices, $d(t) \in R^p$ is the function presenting faults: if faults are absent, $d(t) = 0$, when a fault occurs, $d(t)$ becomes an unknown function of time; the functions $\varphi_1, ..., \varphi_q$ may be non-smooth, $A_1, ..., A_q$ are row matrices. The model (1) can be obtained from general nonlinear system

$$x(t+1) = f(x(t), u(t), d(t)), \qquad y(t) = h(x(t))$$

by special change of coordinates [13]. Specifically, the part of system, presented by constant matrices F and G, is separated from the nonlinear term which is described by nonlinear functions $\varphi_1, ..., \varphi_q$ and matrices C, $A_1, ..., A_q$.

It is assumed that the faults are detected and isolated by methods suggested in [1, 5]. When a fault arises, $d(t)$ is considered as unknown function of time therefore the control problems for system (1) cannot be solved immediately. To deal with the difficulty, we use a feedback described by

$$\begin{aligned} &x_0^+ = F_0 x_0 + G_0 u + J_0 y + C_0\begin{pmatrix} \varphi_1(A_{01} z_0, u) \\ \cdots \\ \varphi_q(A_{0q} z_0, u) \end{pmatrix}, \\ &u = g(x_0, y, u_*), \end{aligned} \tag{2}$$

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where the vector u_* is a new control, $x_0 \in R^{n_0}$, $n_0 \le n$, is the vector of state of the compensator, F_0, G_0, J_0, C_0, A_{01}, ..., A_{0q} are matrices to be determined, $z_0 = \begin{pmatrix} x_0 \\ y \end{pmatrix}$. For simplicity, the notations x_0^+ is used for $x_0(t+1)$.

The goal of decoupling is removing the influence of the unknown function $d(t)$ on a subsystem of maximal dimension of the closed-loop system (1), (2). Specifically, we are looking for system (2) and the subsystem S_* of system (1), (2) of the dimension n_*

$$x_*^+ = F_* x_* + G_* u_* + C_* \begin{pmatrix} \varphi_1(A_{*1} x_*, u_*) \\ \cdots \\ \varphi_q(A_{*q} x_*, u_*) \end{pmatrix}, \qquad (3)$$

which is independent of the unknown function $d(t)$, where $n_* \le n$ is as large as possible and

$$x_*(t) = \Phi x(t) \qquad (4)$$

for some matrix Φ. Since (3) does not depend on the unknown function $d(t)$, one may find the control for subsystem (3) and solve the PRP. Note that since $n_* \le n$, the fault effects can be eliminated only for some function of the state vector $x(t)$ which can be found.

Similar to [4], in the present paper the faults are interpreted as disturbances and the DDP solution of is used for solving the PRP. The disturbance decoupling problem under the compensator (2) is stated as follows [3, 4]. System (1) is considered together with the output-to-be-controlled $y_* \in R^L$, $y_* = h_*(x)$, where h_* is known function. The DDP is design of system (2) in such a way that the variable $y_*(t)$, for $t \ge 0$, of system (1), (2) is not influenced by $d(t)$.

To solve the DDP [3, 4], one finds first a vector function α^0 with maximal number of independent components such that the function $\alpha^0(f(x(t), u(t), d(t)))$ is independent of the unknown function $d(t)$. It can be shown that $\alpha^0(x) = D^0 x$, where D^0 is the matrix of maximal rank such that $D^0 D = 0$.

Recall briefly main definitions and results from [4].

One says that the function α is (h, f)-invariant (f-invariant) if $\alpha(f(x,u,d)) = f_*(\alpha(x), h(x), u, d)$ ($\alpha(f(x,u,d)) = f_*(\alpha(x), u, d)$) where f_* is some function. The function χ is a controlled invariant if a regular static state feedback $u = g'(x, u_*)$ exists such that the function χ in system (1), (2) is f-invariant.

Theorem 1 [4]. The output-to-be-controlled $y_* = h_*(x)$ may be decoupled from the disturbance by compensator (2) if and only if (h, f)-invariant function α and a controlled invariant function χ exist satisfying the condition

$$\alpha^0 \le \alpha \le \chi \le h_*, \qquad (5)$$

where $\beta \le \gamma$ signifies that there exits the function δ satisfying the equality $\delta(\beta(x)) = \gamma(x)$ for all x [3, 10].

III. DDP Solution

A. Preliminaries

Initially we assume that h_* is the known function and design the functions α and χ. Then these results are used for solving the PRP. To simplify a solution, we assume that the functions α and χ are linear ones. This allows using the linear algebra methods for solving the problem under consideration for system (1).

Initially we consider the case when $q = 1$ and construct the system S_0. To construct this system, so-called logic-dynamic approach (LDA) is used which has the following steps [13].

Step 1. Remove the nonlinear part from the initial nonlinear system (1).

Step 2. Solve the considered problem for the linear part, obtained in Step 2, under some linear restriction. Such a restriction is used to know whether or not the nonlinear term can be constructed based on the solution obtained for the linear system.

Step 3. Supplement the obtained in Step 2 solution by the transformed nonlinear term.

In [4], the function α is found as a function having maximal number of components and satisfying the condition $\alpha^0 \le \alpha$. Since it is found as a linear function, one assumes that

$$x_0(t) = \alpha(x(t)) = \Phi x(t) \qquad (6)$$

for some matrix Φ of maximal rank satisfying the following conditions [11, 13]:

$$\Phi F = F_0 \Phi + J_0 H, \quad G_0 = \Phi G, \quad \Phi D = 0. \qquad (7)$$

One can show that the relations $C_0 = \Phi C$ and

$$A = A_0 \begin{pmatrix} \Phi \\ H \end{pmatrix}, \qquad (8)$$

corresponding to the nonlinear term, hold [11, 13].

The last relation is the additional restriction on the matrix Φ mentioned in Step 2. It is true if and only if rows of the matrix A linearly depend on the rows of the matrices Φ and H. Clearly, it is equivalent to the condition

$$rank(\Phi^T \;\; H^T) = rank(\Phi^T \;\; H^T \;\; A^T). \qquad (9)$$

If $q > 1$, the matrix A in (8) and (9) should be substituted for A_i, $i = 1,...,q$.

It is assumed that the matrices F_0 and H_0 are found as

$$F_0 = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \ddots & \cdots \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix}, \quad H_0 = (1 \quad 0 \quad 0 \quad \cdots \quad 0).$$

Then the equation $\Phi F = F_0\Phi + J_0 H$ can be replaced by k equations:

$$\Phi_i F = \Phi_{i+1} + J_{0i}H, \quad i = 1,...,k-1, \quad \Phi_k F = J_{0k}H. \quad (10)$$

The i-th rows of the matrices Φ and J_0 are denoted as Φ_i and J_{0i} are, respectively, $i = 1,...,k$, k is the dimension of x_0.

B. Full Decoupling for Linear System

There exist two ways to find the matrix Φ of maximal rank satisfying the condition $\Phi D = 0$.

1) The first way: It was shown in [13] that (10) with the condition $\Phi D = 0$ can be transformed into the equation

$$(\Phi_1 \quad -J_{01} \quad -J_{02} \; ... \; -J_{0k})(W^{(k)} \quad B^{(k)}) = 0, \quad (11)$$

where

$$W^{(k)} = \begin{pmatrix} F^k \\ HF^{k-1} \\ \cdots \\ H \end{pmatrix}, \quad B^{(k)} = \begin{pmatrix} D & FD & \cdots & F^{k-1}D \\ 0 & HD & \cdots & HF^{k-1}D \\ \cdots & \cdots & \ddots & \cdots \\ 0 & 0 & \cdots & 0 \end{pmatrix}.$$

To achieve the maximal dimension of the system S_0, take $k := n - p$ and check the inequality

$$rank(W^{(k)} \quad H^{(k)}) < lk + n. \quad (12)$$

When (12) holds, there exists the row $(\Phi_1 \quad -J_{01} \; ... \; -J_{0k})$ such that (11) can be solved. Then one finds the matrix Φ from (10) and computes $G_0 = \Phi G$. Thus, the linear part of the system S_0 decoupled from the unknown function $d(t)$ has been constructed.

If (12) does not hold, take $k := k-1$ and continue to check (12). If (12) does not hold for all k, then the system S_0, decoupled from $d(t)$, does not exist and the PRP has no solution. Because the dimension k is maximal, the function $\alpha(x) = \Phi x$ is the best choice for α in (5).

2) The second way: Because the matrix D^0 has full rank such that $D^0 D = 0$ and $\Phi D = 0$, then $\Phi = QD^0$ for some matrix Q. Rewrite the equation $\Phi F = F_0\Phi + J_0 H$ with $\Phi = QD^0$ as

$$(Q \quad -F_0 Q \quad -J_0)((D^0 F)^{\mathrm{T}} \quad (D^0)^{\mathrm{T}} \quad H^{\mathrm{T}})^T = 0. \quad (13)$$

Let $(A \quad B \quad C)$ be a solution of (13), then the equality $B = -F_0 A$ is true. Considering this equality, we remove from the matrix $(A \quad B \quad C)$ all rows which the corresponding rows of B do not depend on the matrix A rows for. Denote the final matrix by $(A^0 \quad B^0 \quad C^0)$. Clearly from (13) that $J_0 = -C^0$ and $\Phi = A^0 D^0$; the matrix F_0 is found from the algebraic equation $B = -F_0 A$.

The first way is good when $n \le 6$; if $n > 6$, the second one is preferable because the matrix $(V^{(k)} \quad H^{(k)})$ has high dimension in this case.

C. Construction of the Dynamic Part of Compensator

Note that if (9) is satisfied for the matrix Φ found for the linear part, then the problem to construct the system S_0 with nonlinearities reduces to that for linear system. When (9) is not satisfied, find all solutions of (11) for maximal k satisfying the condition (12) and present them as

$$(\Phi_1^{(1)} \quad -J_{01}^{(1)} \; ... \; -J_{0k}^{(1)}),...,(\Phi_1^{(N)} \quad -J_{01}^{(N)} \; ... \; -J_{0k}^{(N)}), \quad (14)$$

where the number of all solutions is denoted by N.

Theorem 2 [11]. Consider the matrices $\Phi^{(1)}$, ..., $\Phi^{(N)}$ calculated based on (10) and (11). Then the linear combination of expressions (14) with coefficients v_1, ..., v_N gives the matrix $\Phi = v_1\Phi^{(1)} + ... + v_N\Phi^{(N)}$ which produces some solution of our problem for linear system.

Let the value k is maximal as possible, and all solutions of (11) are found in the form (14). To find the vector $v = (v_1 \; ... \; v_N)$, represent (8) as

$$A = A_{01}\Phi + A_{02}H, \quad (15)$$

where $A_0 = (A_{01} \quad A_{02})$, $A_{01} = (a_1 \; ... \; a_k)$. Denote

$$\Phi_1^{\Sigma} = \begin{pmatrix} \Phi_1^{(1)} \\ \cdots \\ \Phi_1^{(N)} \end{pmatrix}, \; \ldots, \; \Phi_k^{\Sigma} = \begin{pmatrix} \Phi_k^{(1)} \\ \cdots \\ \Phi_k^{(N)} \end{pmatrix}, \quad \Phi^{\Sigma} = \begin{pmatrix} \Phi_1^{\Sigma} \\ \cdots \\ \Phi_k^{\Sigma} \end{pmatrix},$$

and present (15) in the form

$$A = A_{01}((\Phi_1^{\Sigma})^{\mathrm{T}} \quad \cdots \quad (\Phi_k^{\Sigma})^{\mathrm{T}})^{\mathrm{T}} + A_{02}H. \quad (16)$$

Analogously to (9), equation (16) has a solution if

$$rank((\Phi^{\Sigma})^{\mathrm{T}} \quad H^{\mathrm{T}}) = rank((\Phi^{\Sigma})^{\mathrm{T}} \quad H^{\mathrm{T}} \quad A^{\mathrm{T}}). \quad (17)$$

Assume that (17) holds and consider firstly the case when the matrix A has only one row. Here, (16) can be rewritten as $A = (a_1 v \; ... \; a_k v) + A_{02}H$, or as

$$A = A_v \Phi^{\Sigma} + A_{02} H , \qquad (18)$$

where A_v is assumed to be unknown matrix. Solve (18) and find the matrices A_v and A_{02}. If A_v can be rewritten in the form $(a_1 v \ \dots \ a_k v)$ for some coefficients $a_1, \dots, a_k$ and the vector $v = (v_1 \ \dots \ v_N)$, then stop, the matrices A_{01} and A_{02} and the vector $v = (v_1 \ \dots \ v_N)$ have been obtained. Then one finds the rows of J_0 and Φ by

$$J_{0j} = \sum_{i=1}^{N} v_i J_{0j}^{(i)}, \quad \Phi_j = \sum_{i=1}^{N} v_i \Phi_j^{(i)}, \quad j = 1,\dots,k ;$$

$G_0 = \Phi G$, $C_0 = \Phi C$. As a result, a dynamic part of the compensator (2) has been built.

If (17) is not true or the matrix A_v cannot be rewritten in the form $(a_1 v \ \dots \ a_k v)$, one has to decrease the dimension k and repeat the described procedure.

If the number of the matrix A rows is more than one, one solves (18) for each row with coefficients $a_1, \dots, a_k$ peculiar to the considered row; note that the vector v is identical for all rows.

D. Design the Function χ

Recall some results from [4]. Let $h_* = (h_{*1} \ \dots \ h_{*L})^{\mathrm{T}}$; let r_i and w_i be relative degrees of the output $y_{*i} = h_{*i}(x)$ with respect to $u(t)$ and $d(t)$, respectively. Besides, denote $y_{*i}(t) = h_{*i}(x(t)) =: h_{*i,1}(x(t)), \dots, y_{*i}(t + r_i - 1) =: h_{*i,r_i}(x(t))$, $i = 1,\dots,L$. When $h_*(x) = H_* x$, the last relations reduce as follows.

Introduce the matrix C^*: if $C(i,j) \neq 0$ and the function φ_j contains components of the control vector u, set $C^*(i,j) = 1$, otherwise $C^*(i,j) = 0$.

Denote by r_i' minimal integer p such that $H_{*i} F^{p-1} G \neq 0$, by w_i minimal integer p such that $H_{*i} F^{p-1} D \neq 0$, and by $r_i{*}$ minimal integer p such that $H_{*i} F^{p-1} C^* \neq 0$, $i = 1,\dots,L$. Clearly, r_i' and $r_i{*}$ are the relative degrees of the output y_{*i} with respect to $u(t)$ corresponding to the linear and nonlinear parts of system (1), respectively. Set $r_i := \min(r_i', r_i{*})$, $i = 1,\dots,L$.

One makes the following assumptions.

Assumption 1. $w_i > r_i$ and $w_i > r_i'$ for all $i = 1,\dots,L$, otherwise the DDP is not solved.

From the definition of r_i and Assumption 1, $y_{*i}(t + r_i) = \hat{f}_i(x(t), u(t))$ for some function $\hat{f}_i$. Clearly, the function $\hat{f}_i(x(t), u(t))$ is independent of $d(t)$ due to Assumption 1. Assume that $L \le m$ is true and set $\hat{f}(x,u) := (\hat{f}_1(x,u), \dots, \hat{f}_L(x,u))^{\mathrm{T}}$.

Vector $(r_1, \dots, r_L)$ is said to be vector relative degree of the output-to-be-controlled $y_*(t)$ if the equality $rank(\partial \hat{f}(x,u)/\partial u) = L$ holds everywhere except perhaps on a set of measure zero.

Assumption A2. The output-to-be-controlled $y_*(t)$ has a vector relative degree $(r_1, \dots, r_L)$.

Theorem 3 [4]. Under Assumptions 1 and 2, the controlled invariant function χ satisfying the inequality $\chi \le h_*$ and having minimal number of components, can be computed by

$$\chi := ((h_{*1}^{0})^{\mathrm{T}} \ \dots \ (h_{*L}^{0})^{\mathrm{T}})^{\mathrm{T}}, \qquad (19)$$

where $h_{*i}^{0} = (h_{*i,1} \ \dots \ h_{*i,r_i})$, $i = 1,\dots,L$.

We would like to find the function χ as a linear one, so the additional assumption is stated.

Assumption 3. $r_i = r_i'$ for all $i = 1,\dots,L$, i.e. all relative degrees correspond to the linear part of system (1).

Set $y_{*1} = H_{*1} x$, …, $y_{*1}^{r_1} = H_{*1} F^{r_1 - 1} x$; clearly, the output

$$\begin{aligned} y_{*1}^{r_1 +} &= H_{*1} F^{r_1 - 1} x^{+} = H_{*1} F^{r_1 - 1} (Fx + Gu + \Psi(x,u)) \\ &= H_{*1} F^{r_1} x + H_{*1} F^{r_1 - 1} G u + \psi_1(x) \end{aligned}$$

depends on control. Here $\psi_1(x) = H_{*1} F^{r_1 - 1} \Psi(x,u)$; note that the function $\psi_1(x)$ is independent of u due to Assumption 3. Clearly, the expression $H_{*1} F^{r_1} x + H_{*1} F^{r_1 - 1} G u + \psi_1(x)$ is similar to the function $\hat{f}_1(x,u)$.

Consider the set of equations

$$\begin{aligned} &H_{*1} F^{r_1} x + H_{*1} F^{r_1 - 1} G u + \psi_1(x) = u_{*1}, \\ &\dots \\ &H_{*L} F^{r_L} x + H_{*L} F^{r_L - 1} G u + \psi_L(x) = u_{*L}. \end{aligned} \qquad (20)$$

Introduce the matrices

$$H_*^{(i)} = \begin{pmatrix} H_{*i} \\ \cdots \\ H_{*i} F^{r_i - 1} \end{pmatrix}, \quad i = 1,\dots,L, \quad \hat{H}_* = \begin{pmatrix} H_{*1} F^{r_1 - 1} G \\ \cdots \\ H_{*L} F^{r_L - 1} G \end{pmatrix}. \qquad (21)$$

Assume for simplicity that $rank(\hat{H}_*) = L$; clearly, this condition is equivalent to Assumption 2. In this case (20) can be solved for the control u.

Set $\Phi_* = ((H_*^{(1)})^{\mathrm{T}} \ \dots \ (H_*^{(L)})^{\mathrm{T}})^{\mathrm{T}}$. Clearly, the matrix Φ_* is similar to the function χ defined by (19). As a result, this matrix can be considered as controlled invariant for the linear part of the closed-loop system (1), (2). If the condition

$$rank(\Phi_*) = rank(\Phi_*^{\mathrm{T}} \quad A^{\mathrm{T}}) \tag{22}$$

is true then the nonlinear term of the system S_* can be constructed based on the linear part. Analogue of the condition $\alpha \le \chi$ is the equality

$$rank(\Phi) = rank(\Phi_*^{\mathrm{T}} \quad \Phi^{\mathrm{T}}). \tag{23}$$

If (22) and (23) are true, the PRP is solvable; otherwise a solution does not exist. Assuming that (22) and (23) are true, we have $\Phi_* = Q\Phi$ for some matrix Q.

Solving (20) for the control, one obtains the expression in the form $u = g'(x, u_*)$, i.e. a static state form of the feedback. Since $\Phi_* = Q\Phi$ and the matrix Φ is similar to the (h, f)-invariant function, then the state x in $u = g'(x, u_*)$ can be expressed in the terms of the state $x_0 = \Phi x$ and the vector y. As a result, a static part of the compensator (2) takes the form $u = g(x_0, y, u_*)$ for some function g.

The matrix Φ_* is used to construct the system S_*: set $x_* = \Phi_* x$, then $x_*^+ = \Phi_* x^+ = \Phi_* Fx + \Phi_* Gu + \Phi_* \Psi(x, u)$. Since Φ_* is controlled invariant for the linear part of the closed-loop system and (23) is valid, the right-hand side of the last expression can be expressed via the state x_* and the new input u_* i.e. one obtains the expressions in the form (3).

If the condition $r_i = r_i'$ is not true for some i, then the function ψ_i in the left-hand sides in (20) depends on the control u. In this case the matrix Φ_* remains the analogue of χ, the expressions for g' and g become more complex [12].

IV. Solution of the PRP

Here, we use the DDP solution for solving the PRP. Note that when the PRP is stated, the matrix H_* is a design object unlike the DDP where H_* is given. So, we find a matrix H_* of maximal rank which is independent of $d(t)$ using the compensator (2).

We take $H_* = \Phi$ since this is the best choice for H_* due to Theorem 1 and find out whether or not the DDP can be solved for $H_* = \Phi$. Due to Theorem 1, a controlled invariant function χ exist such that $\alpha \le \chi \le h_*$. Taking into account our analogues, one has $H_* = \Phi$ and the only possible choice for Φ_* is $\Phi_* := \Phi$. Hence, we have to check whether Φ is controlled invariant. If yes, the best solution of the PRP has been obtained where the system S_* has maximal dimension. Otherwise the PRP has a solution where the system S_* is of smaller dimension. The matrix Φ_* of such a solution is satisfied the equality $\Phi_* = Q\Phi$. In this case $H_* := \Phi_*$.

An algorithm computing the feedback that solves the PRP is given below.

Algorithm.

Step 1. Design the dynamic part of the feedback (2) based on Section III and take $H_* := \Phi$. Assume that the condition (9) is satisfied.

Step 2. Find the relative degrees r_i', $r_i{*}$, and w_i of $y_{*i} = H_{*i} x$. Assuming that $r_i = r_i'$, check Assumption 1; if it does not hold, remove the i-th row from the matrix H_*, $i = 1,..., L$. Denote the final matrix by H_* as well.

Step 3. Compute the matrix $H_*^{(i)}$ from (21) and check the condition

$$rank(\Phi) = rank((H_*^{(i)})^{\mathrm{T}} \quad (\Phi_*)^{\mathrm{T}}). \tag{24}$$

If it is not true, then remove the i-th row from the matrix H_*, $i = 1,..., L$. Denote the final matrix by H_* as well.

Step 4. Compute the matrix Φ_*; clearly, the condition (24) is satisfied. Check (23); if it is not true, then stop, the PRP is not solved.

Step 5. Construct the matrix $\hat{H}_*$ from (21) and check the condition $rank(\hat{H}_*) = L$. If it is satisfied, go to Step 7, otherwise to Step 6.

Step 6. If $rank(\hat{H}_*) = M < L$, then find $M \times L$ matrix P satisfying the condition $rank(P\hat{H}_*) = M$. The matrix P collects the linearly independent rows of $\hat{H}_*$. Set $\hat{H}_* := P\hat{H}_*$.

Step 7. Design the static part of (2) by solving (20) for u in the form $u = g'(x, u_*)$ and next in the form $u = g(x_0, y, u_*)$ (when $M < L$, the symbol L in (20) is substituted for M).

Step 8. Set $H_* := \Phi_*$, $x_* := \Phi_* x$, construct the system $x_*^+ = \Phi_* x^+ = \Phi_* Fx + \Phi_* Gu + \Phi_* \Psi(x, u)$, and transform it into the form (3). Take $y_* = H_* x = x_*$.

Since the matrix P in Step 6 can be chosen by several ways, it should be chosen in such a way that the relative degree of $y_* = H_* x = x_*$ is maximal.

V. Example

Consider the system described by the equations

$$\begin{aligned}
&x_1^+ = x_3 + x_6 + x_4 + u_3 + d_1, && x_2^+ = \mathrm{sign}(x_3) + x_6 + u_1,\\
&x_3^+ = -x_3 x_4, && x_4^+ = x_4 + x_5 + u_1,\\
&x_5^+ = x_3 + x_4 + d_2, && x_6^+ = x_2^2 + x_1 + u_2,\\
&y_1 = x_1, \quad y_2 = x_5. &&
\end{aligned}$$

As recommended in [11], the initial model is corrected by entering several formal terms as follows: the term $x_3 - x_3$ in the second equation, $x_3 + x_4 - x_3 - x_4$ in the third, and $x_2 - x_2$ in the fifth. The resulting matrices and nonlinearities are as follows:

$$F = \begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad G = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix},$$

$$H = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad D = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix},$$

$$\varphi_1(x,u) = \mathrm{sign}(x_3) - x_3, \qquad A_1 = (0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0),$$
$$\varphi_2(x,u) = x_3x_4 + x_3 + x_4, \qquad A_2 = (0\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0),$$
$$\varphi_3(x,u) = x_2^2 - x_2, \qquad A_3 = (0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0).$$

Compute

$$D^0 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

It can be shown that $\Phi = D^0$. The condition (9) is satisfied therefore the nonlinear term of (2) can be constructed based on the matrix Φ. Clearly, $C^* = 0$, $H_* = \Phi$, $L = 4$.

Next, by Step 2 of Algorithm find $r_1' = r_3' = r_4' = 1$, $r_2' = 2$, $r_1{}^* = ... = r_4{}^* = \infty$, $w_1 = w_2 = 3$, and $w_3 = w_4 = 2$; clearly, Assumptions 1 and 3 are satisfied.

Clearly, the condition (24) is satisfied for all *i*. It can be shown that (22) is satisfied as well therefore the nonlinear term of the system S_* can be constructed based on the linear part. Compute

$$\hat{H}_* = \begin{pmatrix} H_{*1}F^{r_1-1}G \\ \cdots \\ H_{*L}F^{r_L-1}G \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}.$$

Since $rank(\hat{H}_*) = 2 < 4$, Assumption 2 is not satisfied. By Step 6, find the matrices

$$P = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \hat{H}_* := P\hat{H}_* = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}.$$

Clearly, equations (20) with $\hat{H}_*$ are solvable for u_1, u_2. Set $u_{*1} := x_4 + x_5 + u_1$, $u_{*2} := x_2^2 + x_1 + u_2$. Since $x_0 := \Phi x$, set $(x_{01}, x_{02}, x_{03}, x_{04})^{\mathrm{T}} := (x_2, x_3, x_4, x_6)^{\mathrm{T}}$ and find the system S_0:

$$x_{01}^+ = \mathrm{sign}(x_{02}) + x_{04} + u_1, \quad x_{02}^+ = -x_{02}x_{03},$$
$$x_{03}^+ = x_{03} + y_2 + u_1, \quad x_{04}^+ = x_{02}^2 + y_1 + u_2.$$

In Step 7, we replace (x_2, x_3, x_4, x_6) by $(x_{01}, x_{02}, x_{03}, x_{04})$ and obtain

$$u_{*1} := x_{03} + y_2 + u_1, \quad u_{*2} := x_{02}^2 + y_1 + u_2.$$

As a result, the static part of the compensator (2) is given by

$$u_1 = u_{*1} - x_{03} - y_2, \quad u_2 = u_{*2} - x_{02}^2 - y_1, \quad u_3 = u_{*3}.$$

To construct the system S_*, set $H_* := \Phi_*$, $(x_{*1}, x_{*2}, x_{*3}, x_{*4})^{\mathrm{T}} := (x_2, x_3, x_4, x_6)^{\mathrm{T}}$ and obtain its description in Step 8:

$$x_{*1}(t+1) = \mathrm{sign}(x_{*2}(t)) + x_{*4}(t) + (u_{*1}(t) - x_{*4}(t))/x_{*3}(t),$$
$$x_{*2}(t+1) = -x_{*2}(t)x_{*3}(t),$$
$$x_{*3}(t+1) = u_{*1}(t),$$
$$x_{*4}(t+1) = u_{*2}(t).$$

VI. CONCLUSION

The paper deals with the plant reconfiguration problem. The advantage of the LDA which is used in the paper to solve this problem is that the system can be described by a model with non-smooth nonlinearities. Besides, this approach can be used for the continuous-time systems as well advantageously as for the discrete-time ones.

REFERENCES

[1] S. Ding, Data-driven Design of Fault Diagnosis and Fault-tolerant Control Systems, London: Springer-Verlag, 2014.

[2] B. Jiang, M. Staroswiecki and V. Cocquempot, "Fault accommodation for nonlinear dynamic systems", IEEE Transactions on Automatic Control, vol. 51, pp. 1578-1583, 2006.

[3] A. Kaldmae, U. Kotta, A. Shumsky, and A. Zhirabok, "Disturbance decoupling in nonlinear hybrid systems", Nonlinear Analysis: Hybrid Systems, vol. 28, pp. 42-53, 2018.

[4] A. Kaldmae, U. Kotta, A. Shumsky, and A. Zhirabok, "Faulty plant reconfiguration based on disturbance decoupling methods", Asian Journal of Control, vol. 8, pp. 858-867, 2016.

[5] I. Samy, I. Postlethwaite, and D. Gu, "Survey and application of sensor fault detection and isolation schemes", Control Engineering Practice, vol. 19, 658-674, 2011.

[6] A. Shumsky, A. Zhirabok, and E. Bobko, "Fault accommodation in dynamic systems: logic-dynamic approach, in Diagnosis of Processes and Systems (Ed. Z. Kowalczuk), Gdansk: PWNT, 2009, 141-148.

[7] A. Shumsky, V. Filaretov, A. Zhirabok, A. Zuev, and E. Bobko, "Fault accommodation in technical systems based on logic-dynamic approach", Procedia Engineering, vol. 100, pp. 234–241, 2015.

[8] M. Witczak, Fault Diagnosis and Fault Tolerant Control Strategies for Nonlinear Systems, London: Springer, 2014.

[9] H. Yang, B. Jiang, and V. Cocquempot, Fault Tolerant Control Design for Hybrid Systems, Berlin-Heidelberg: Springer-Verlag, 2010.

[10] K. Zhang and V. Cocquempot, "Fast Adaptive Fault Estimation and Accommodation for Nonlinear Time-Varying Delay Systems", Asian Journal of Control, vol. 11, pp. 643-652, 2009.

[11] A. Zhirabok, A. Shumsky, and E. Bobko, "Fault accommodation in bilinear systems - a logic-dynamic approach", Int. Journal of Quality and Reliability Management, vol. 31, pp.129-143, 2014.

[12] A. Zhirabok, A. Shumsky, A. Zuev, and E. Bobko, "Faulty system reconfiguration: logic-dynamic approach", in 12th IEEE Conf. on Industrial Electronics and Applications, 2017, pp. 1479-1484.

[13] A. Zhirabok, A. Shumsky, S. Solyanik, and A. Suvorov, "Fault detection in nonlinear systems via linear methods", Int. Journal of Applied Mathematics and Computer Science, vol. 27, no. 2, pp. 261-272, 2017.